Sketching sine and cosine

When we sketch a sine or cosine function of the form

$$a\sin(b\theta)$$

$$a\cos(b\theta)$$

where a and b are nonzero real numbers and where b is positive, the first thing we want to do is find the amplitude $|a|$ and period $2\pi/b$ of the function. Then we can plot points where the function is either 0, a, or $-a$. We pick those points because they'll be easy to find, and then we can use those points to sketch the curve.

Sketching sine functions

It's helpful to remember that sine functions in the form $a\sin(b\theta)$ will always equal 0 at $\theta = 0$, and halfway through each period.

Example

Sketch the graph of $3\sin(2\theta)$.

In this case $a = 3$ and $b = 2$, so the amplitude is $|a| = 3$ and the period is

$$\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$$

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So this function is 0 at $\theta = 0$ and halfway through the period at

$$\theta = \frac{\pi}{b} = \frac{\pi}{2}$$

Also, it's equal to $a = 3$ at

$$\theta = \frac{\pi}{2b} = \frac{\pi}{2(2)} = \frac{\pi}{4}$$

and to $-a = -3$ at

$$\theta = \frac{3\pi}{2b} = \frac{3\pi}{2(2)} = \frac{3\pi}{4}$$

Since a is positive, this function is positive in the interval

$$\left(0, \frac{\pi}{2}\right)$$

and negative in the interval

$$\left(\frac{\pi}{2}, \pi\right)$$

Using this information, let's sketch the graph over the interval $[0, \pi)$.

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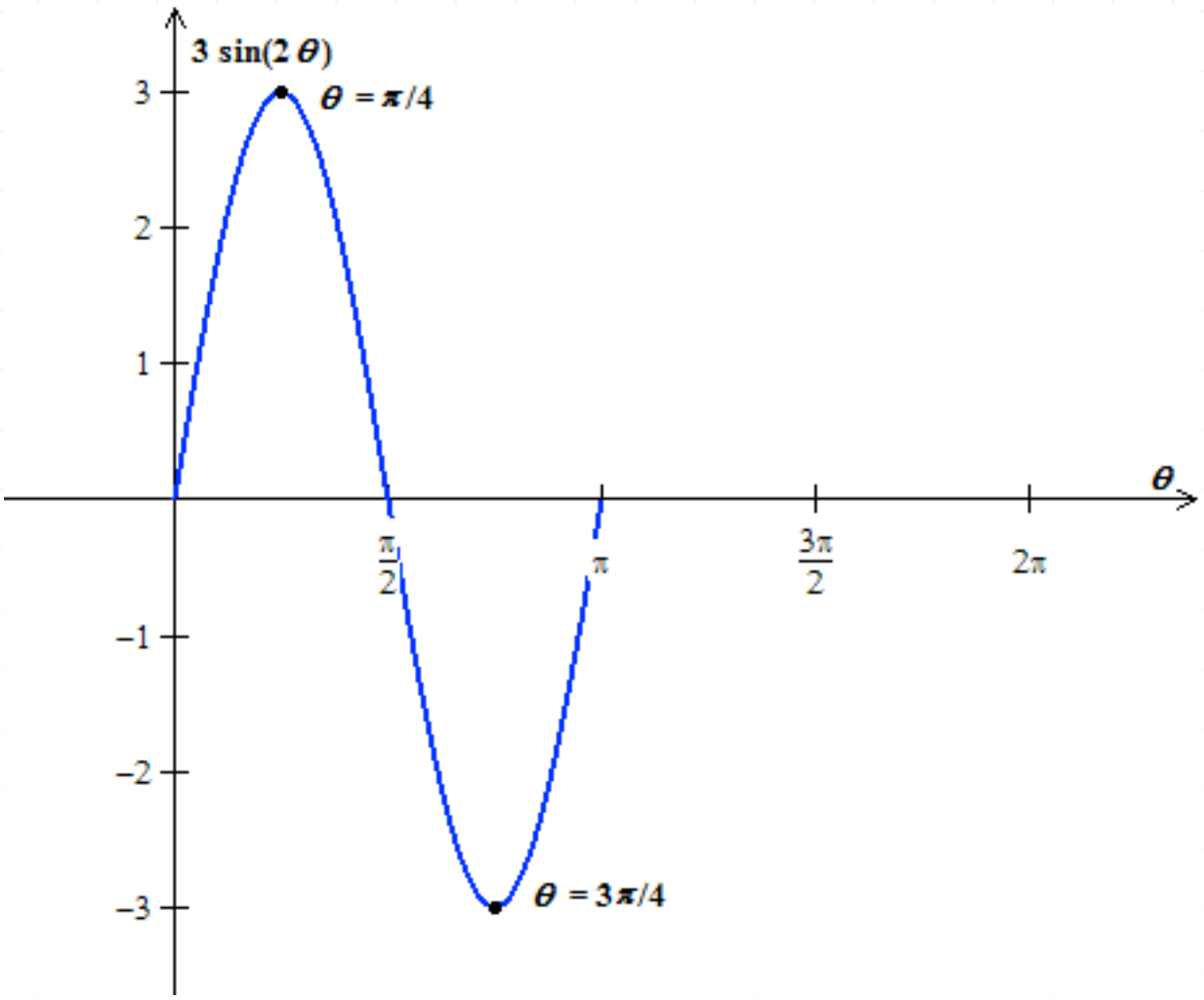
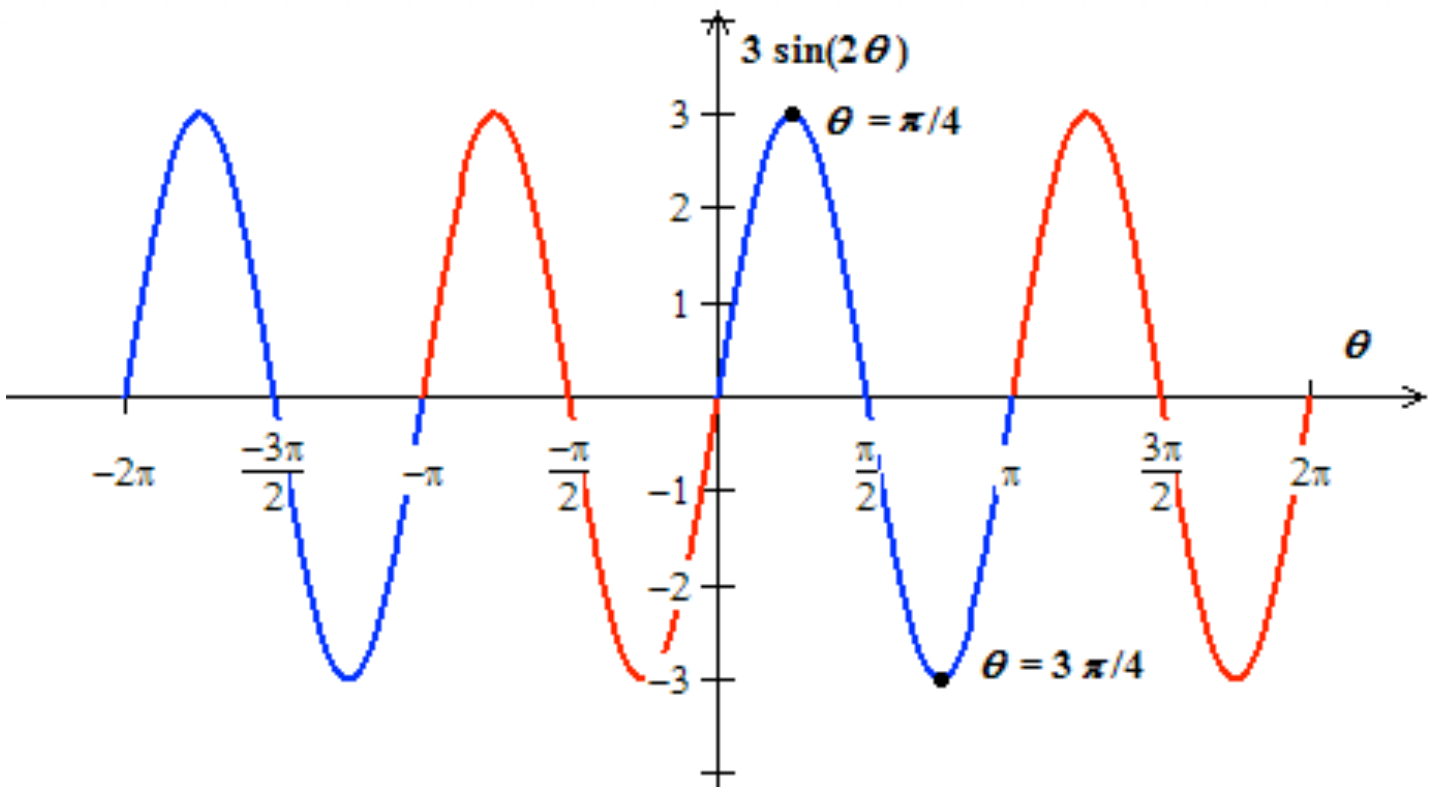


We could also expand the graph to show more periods in both directions.



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Now consider a function of the form $a\sin(b\theta)$ where a is negative. Notice that such a function is negative in the interval

$$\left(0,\frac{\pi}{b}\right)$$

and positive in the interval

$$\left(\frac{\pi}{b},\frac{2\pi}{b}\right)$$

Example

Sketch the graph of $-5\sin(3\theta)$.

Here, $a=-5$ and $b=3$, so the amplitude is $|a|=5$ and the period is

$$\frac{2\pi}{b}=\frac{2\pi}{3}$$

So this function is equal to 0 at $\theta=0$ and halfway through the period at

$$\theta=\frac{\pi}{b}=\frac{\pi}{3}$$

Also, it's equal to $a=-5$ at

$$\theta=\frac{\pi}{2b}=\frac{\pi}{2(3)}=\frac{\pi}{6}$$

and to $-a=5$ at

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$$\theta = \frac{3\pi}{2b} = \frac{3\pi}{2(3)} = \frac{\pi}{2}$$

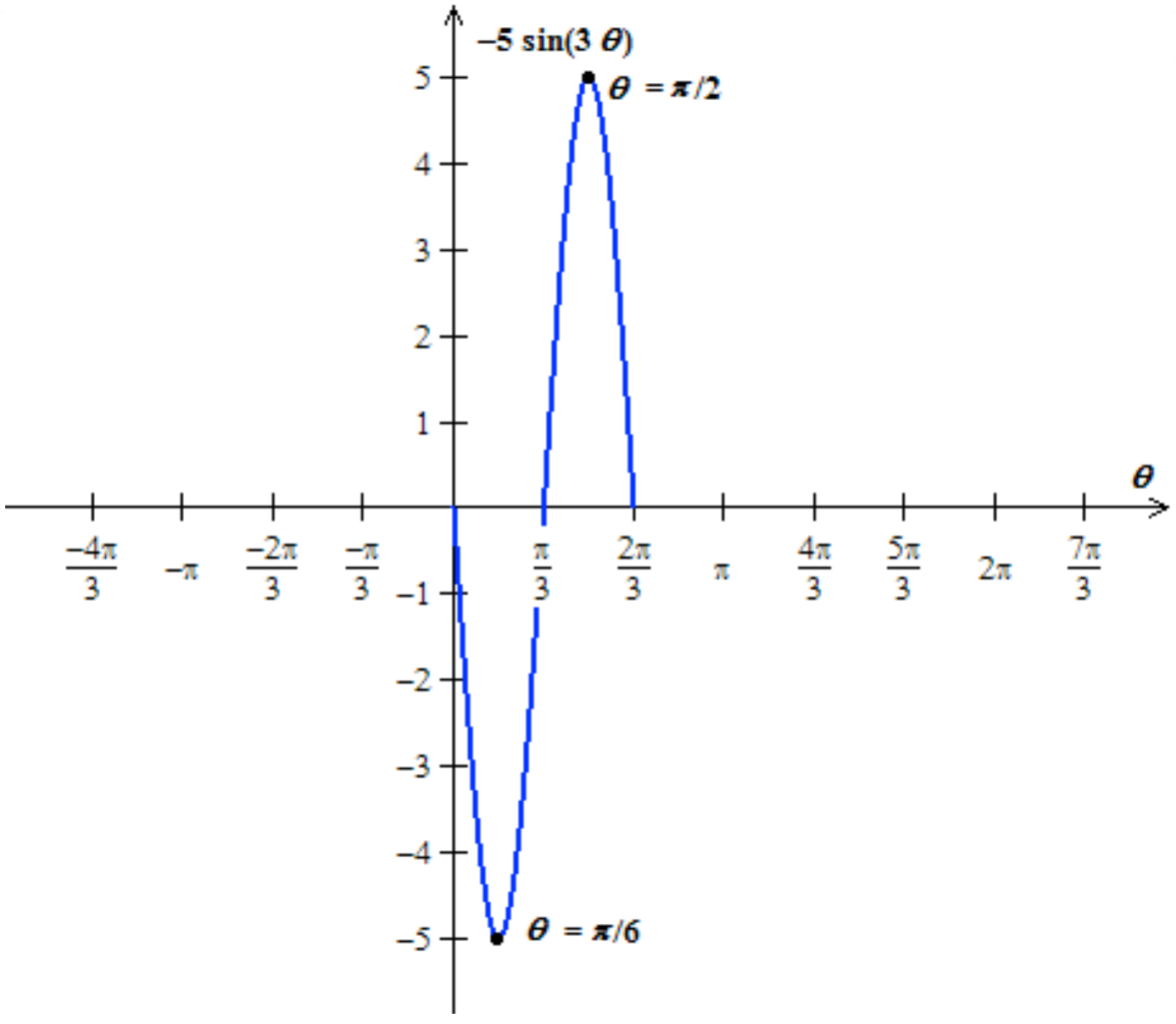
Since a is negative, the value of this function is negative in the interval

$$\left(0, \frac{\pi}{3}\right)$$

and positive in the interval

$$\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$$

Using all of this information, let's sketch the graph over the interval $[0, 2\pi/3)$.



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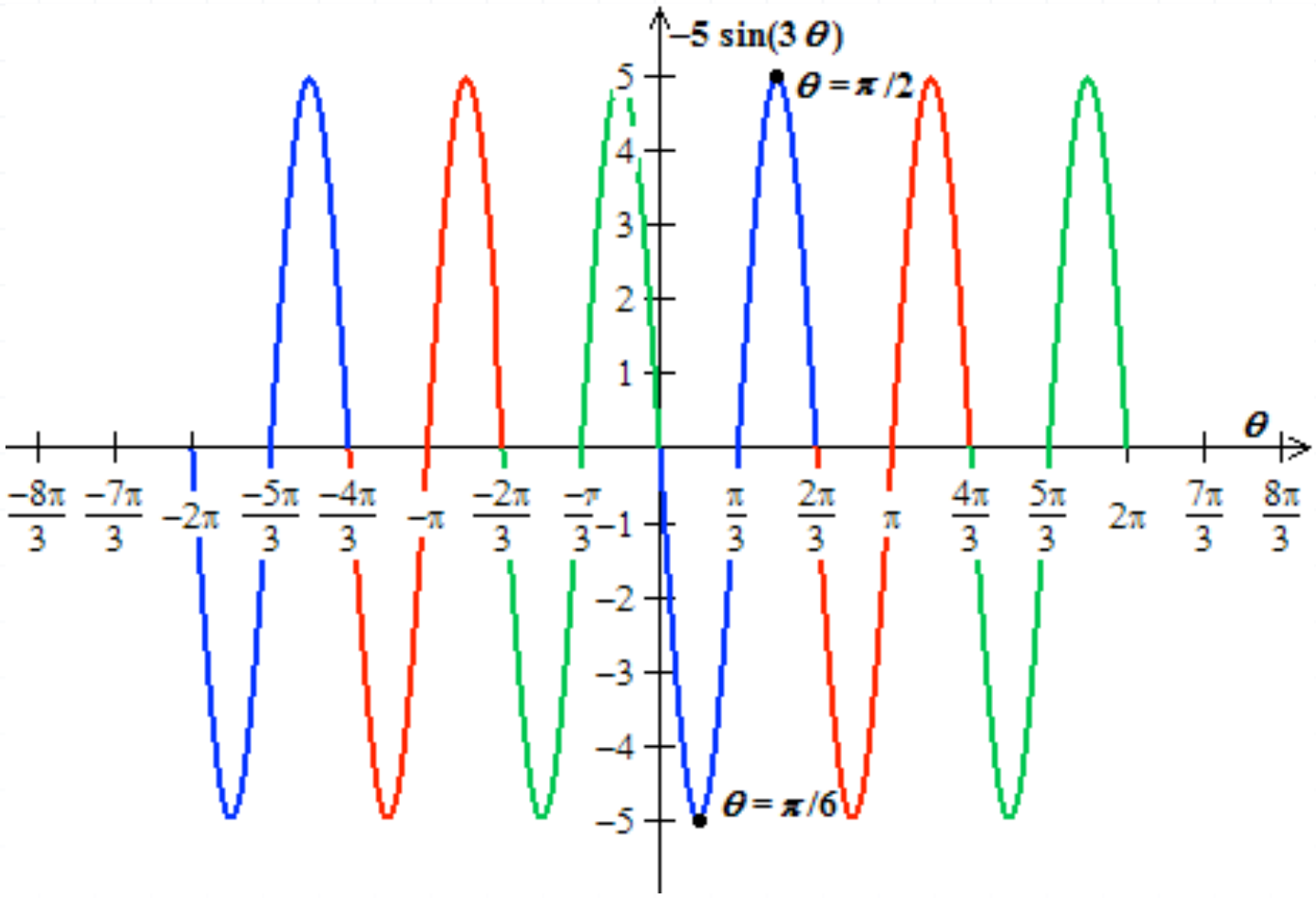
We could also expand the graph to show more periods in both directions.



Remember that exactly b periods of the function $a\sin(b\theta)$ fit into the interval $[0,2\pi)$. So if $b < 1$, then only a fraction of a period fits into the interval $[0,2\pi)$.

Example

Sketch the graph of $3\sin(\theta/2)$.

This function has $a = 3$ and $b = 1/2$, so its amplitude is $|a| = 3$ and its period is

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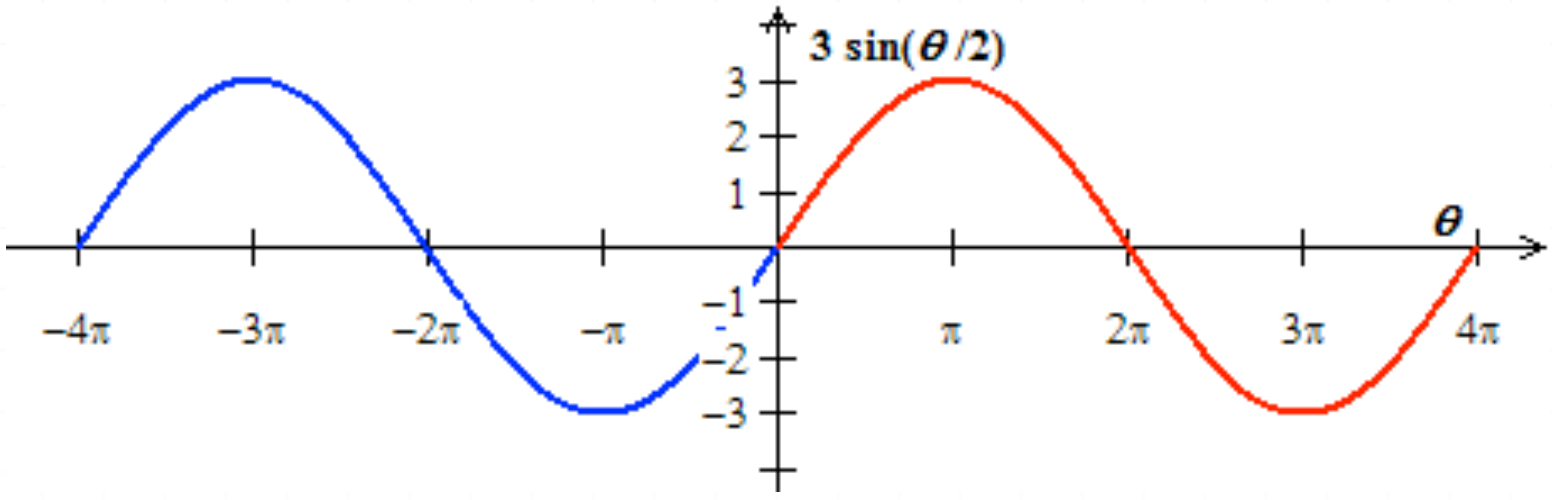
$$\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 2\pi\left(\frac{2}{1}\right) = 4\pi$$

Therefore, exactly half of a period of this function fits into the interval $[0,2\pi)$. Let's sketch this function over two consecutive periods: $[-4\pi,0)$ and $[0,4\pi)$.



Sketching cosine functions

Now let's talk about sketching cosine functions. It's helpful to remember that cosine functions in the form $a\cos(b\theta)$ will always equal 0 one-fourth and three-fourths of the way through each period.

Example

Sketch the graph of $2.6\cos(3\theta)$.

In this case, $a = 2.6$ and $b = 3$, so the amplitude is $|a| = 2.6$ and the period is

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$$\frac{2\pi}{b} = \frac{2\pi}{3}$$

This function is equal to 0 at

$$\theta = \frac{\pi}{2b} = \frac{\pi}{2(3)} = \frac{\pi}{6}$$

and

$$\theta = \frac{3\pi}{2b} = \frac{3\pi}{2(3)} = \frac{\pi}{2}$$

Also, it's equal to $a = 2.6$ at $\theta = 0$, and to $-a = -2.6$ at

$$\theta = \frac{\pi}{b} = \frac{\pi}{3}$$

Since a is positive, this function is positive in the intervals

$$\left(0, \frac{\pi}{6}\right)$$

and

$$\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$$

Also, it's negative in the interval

$$\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$$

Using this information, let's sketch the graph of this function over the interval $[0, 2\pi/3)$.

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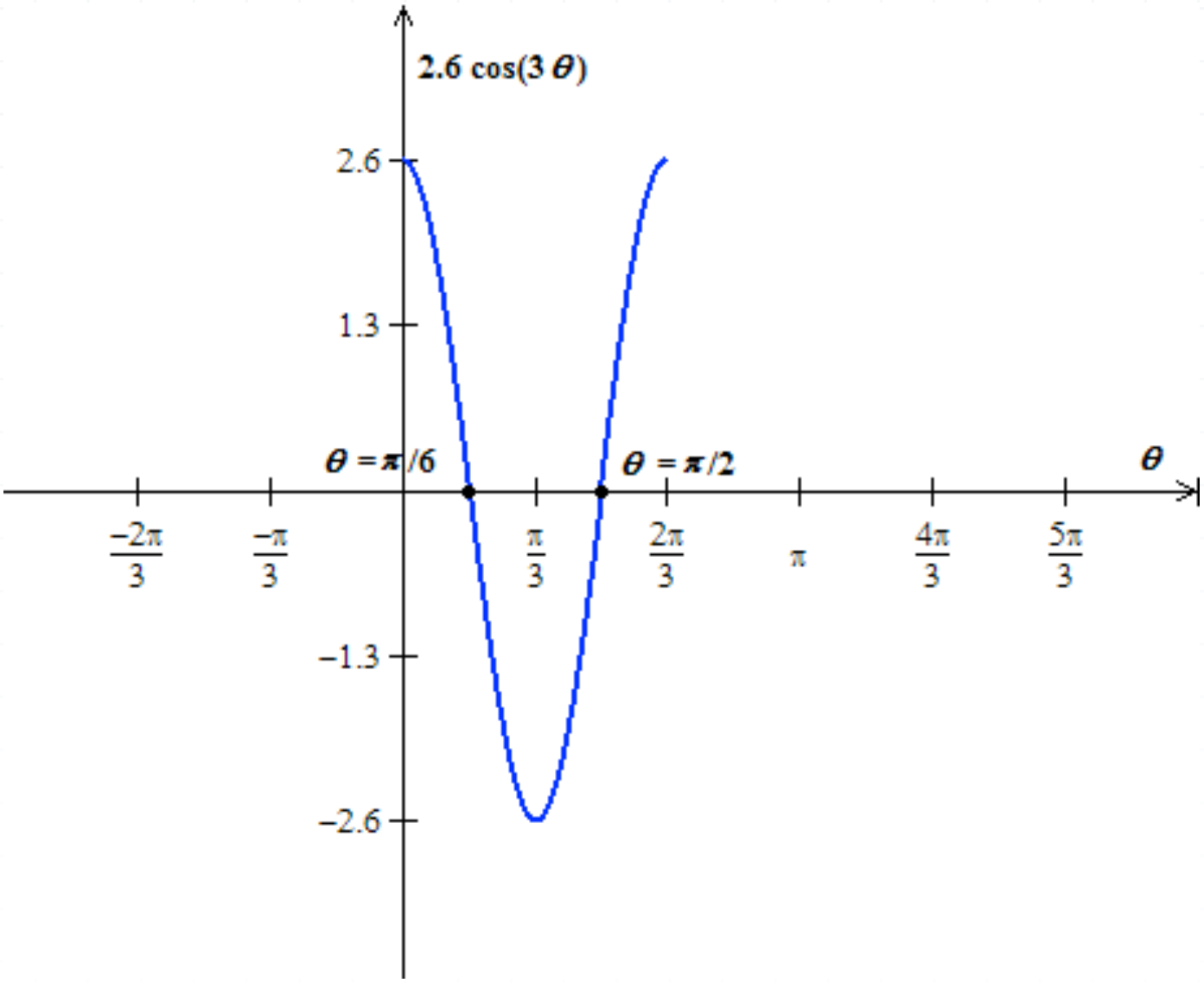
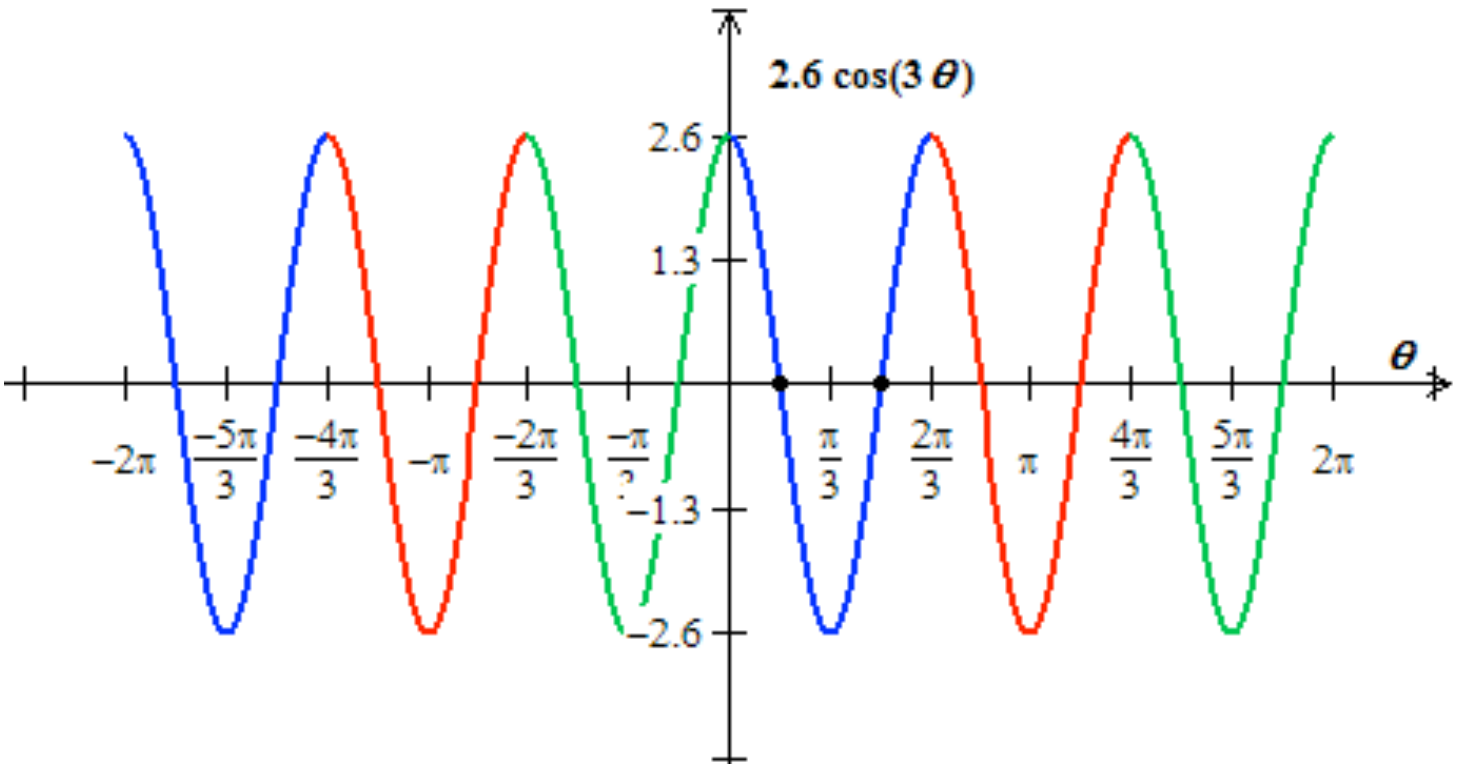


We could also expand the graph to show more periods in both directions.



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Next, we'll consider a function of the form $a\cos(b\theta)$ where a is negative. Notice that such a function is negative in the intervals

$$\left(0, \frac{\pi}{2b}\right)$$

and

$$\left(\frac{3\pi}{2b}, \frac{2\pi}{b}\right)$$

Also, it's positive in the interval

$$\left(\frac{\pi}{2b}, \frac{3\pi}{2b}\right)$$

Example

Sketch the graph of $-4\cos(\theta/3)$.

Here, $a = -4$ and $b = 1/3$, so the amplitude is $|a| = 4$ and the period is

$$\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{3}} = 2\pi\left(\frac{3}{1}\right) = 6\pi$$

This function is equal to 0 at

$$\theta = \frac{\pi}{2b} = \frac{\pi}{2\left(\frac{1}{3}\right)} = \pi\left(\frac{3}{2}\right) = \frac{3\pi}{2}$$

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and

$$\theta = \frac{3\pi}{2b} = \frac{3\pi}{2\left(\frac{1}{3}\right)} = 3\pi\left(\frac{3}{2}\right) = \frac{9\pi}{2}$$

Also, it's equal to $a = -4$ at $\theta = 0$, and to $-a = 4$

$$\theta = \frac{\pi}{b} = \frac{\pi}{\frac{1}{3}} = \pi\left(\frac{3}{1}\right) = 3\pi$$

Since a is negative, this function is negative in the intervals

$$\left(0, \frac{3\pi}{2}\right)$$

and

$$\left(\frac{9\pi}{2}, 6\pi\right)$$

Also, it's positive in the interval

$$\left(\frac{3\pi}{2}, \frac{9\pi}{2}\right)$$

Let's graph this function over two consecutive periods: $[-6\pi, 0)$ and $[0, 6\pi)$.

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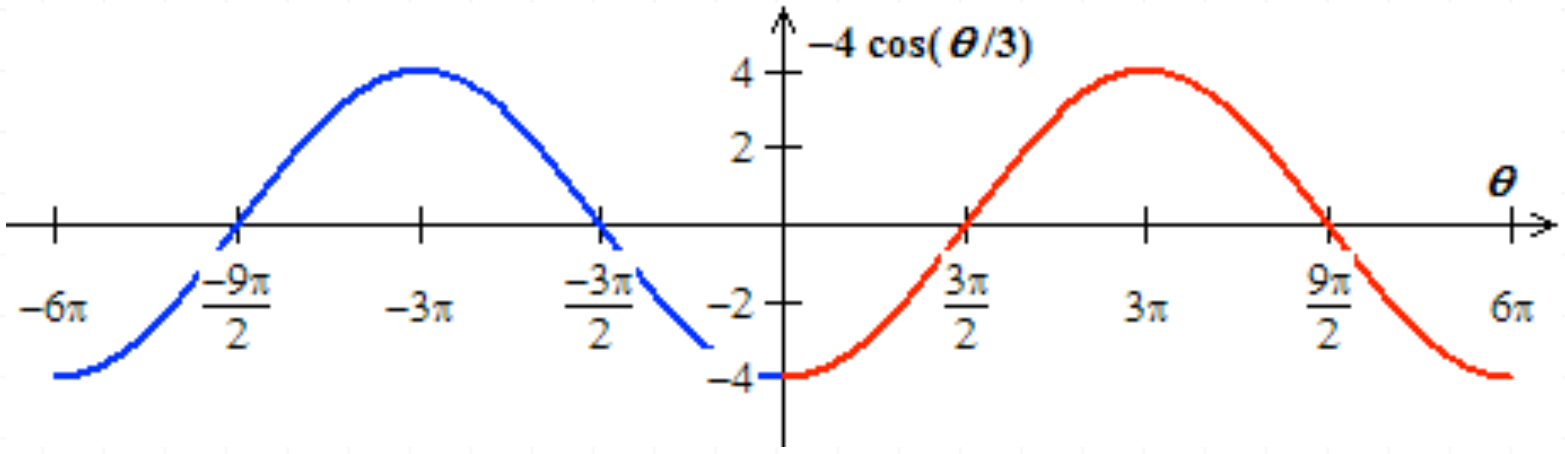

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